Non-universal conductance in quasi-helical quantum wires

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In a quantum wire with ideal helical modes, the conductance is quantized in units of e^2/h, provided the wire is connected to Fermi liquid leads. We show that this universality does not hold in partially gapped quasi-helical systems such as Rashba nanowires subject to a magnetic field, which are commonly used to mimic helical Luttinger liquids. Instead, their conductance takes a non-universal value that depends on the interactions in the wire, even in the presence of Fermi liquid leads. The non-universal conductance is rooted in a non-trivial mixing of spin and charge degrees of freedom, which in turn defines a non-helical low-energy Hamiltonian.

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The conductance of a nanowire is an important example for quantization effects in mesoscale and nanoscale structures. Since its experimental detection in quantum point contacts, [1, 2] conductance quantization has become a versatile diagnostics for low-dimensional quantum physics. The robustness of this effect, in particular in quantum Hall samples, even allows for the definition of highest accuracy standards of the electric resistance. [3, 4] In an infinite wire, the conductance G depends on electron-electron interactions and is thus non-universal. [5, 6] Once the (gapless) wire, however, is connected to Fermi liquid leads, G becomes independent of interactions. [7–9] Each channels then contributes a single quantum $G_0 = e^2/h$ to the overall transport. A conductance close to G_0 has also been observed in helical Luttinger liquids, experimentally for instance realized as edge channels of quantum spin Hall systems, [10] in which the spin of the electrons is locked to their direction of motion. In the remainder, we contrast the unit conductance of ideal helical Luttinger liquids to the case of quasi-helical Luttinger liquids, where the conductance becomes non-universal even in the presence of Fermi liquid leads. Experimental examples for quasi-helical systems include Rashba spin-orbit coupled quantum wires (Rashba nanowires) in a homogeneous magnetic field, [11] see Fig. 1, which are formally equivalent to wires without spin-orbit coupling in a helical field. [12] Rashba nanowires are for instance important in the context of Majorana bound states in topological quantum wire. [13] Quasi-helical regimes can also be found in nanowires in a spatially oscillating (rotating) magnetic field, [14], in Carbon nanotubes, [15] or wires in which the electron spins are coupled to a Kondo lattice in the RKKY regime, such as the nuclear spins. [16, 17] Besides the conductance analyzed in this work, it has also been argued that the spectral density and optical conductivity of partially gapped quasi-helical wires, [18, 19] as well as their response to disorder, [20] differ from the ones of ideal helical Luttinger liquids. In view of these differences, the former systems have been dubbed spiral Luttinger liquids or spiral spin density wave states. Since helical Luttinger liquids can be understood as a special subclass of spiral

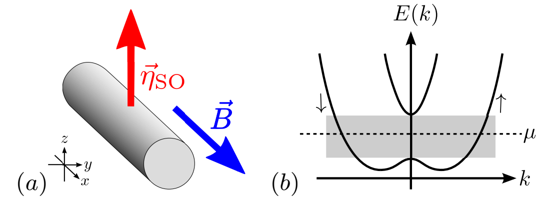

FIG. 1: Panel (a) depicts an example for a quasi-helical system, namely a nanowire with Rashba spin-orbit coupling $\sim \boldsymbol{\eta}_{\rm SO} \cdot \boldsymbol{S}$ in a magnetic field $\boldsymbol{B} \perp \boldsymbol{\eta}_{\rm SO}$ ($\boldsymbol{S}$ is the electron spin). Panel (b) shows the wire's spectrum $E(k)$. The shaded quasi-helical regime, exhibiting a quasi-helical spin polarization, can be reached by adjusting the chemical potential μ.

ones with Luttinger liquid parameters $K_c = 1/K_s$, [18] it is not surprising that general spiral Luttinger liquids show physics beyond the helical limit. In this latter limit, our analysis however recovers the universal value $G = G_0$. In the following, we argue that the non-universal value of the conductance results from a non-trivial mixing of spin and charge degrees of freedom. To this end, we show that the definition of the remaining gapless modes does not straightforwardly follow from the form of the gapped ones, but has to be inferred from a renormalization group (RG) argument. Our considerations apply quite generally when the strong coupling theory resulting from gapping out a combination of fields in a quantum wire is needed for the calculation of physical observables.

We analyze the following model of a spiral Luttinger liquid of length L connected to Fermi liquid leads, expressed in bosonic spin and charge fields, [21]

$$\begin{aligned} H_W = &\frac{1}{2\pi}\int_{-L/2}^{L/2} dx \left(\frac{u_c}{K_c}(\partial_x\phi_c)^2 + u_c K_c(\partial_x\theta_c)^2\right) \\ &+ \frac{1}{2\pi}\int_{-L/2}^{L/2} dx \left(\frac{u_s}{K_s}(\partial_x\phi_s)^2 + u_s K_s(\partial_x\theta_s)^2\right) \\ &+ \frac{B}{2\pi\alpha}\int_{-L/2}^{L/2} dx\, \cos\left(\sqrt{2}(\phi_c+\theta_s)\right) \ . \end{aligned} \quad (1)$$

Details on the derivation of this model may be found elsewhere. [12, 16, 17] As usual, K_c (K_s) is the Lut-

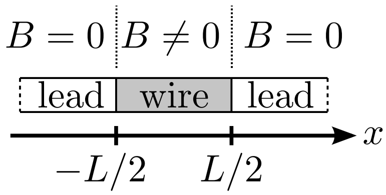

FIG. 2: Sketch of the model: an interacting wire subject to a field B is sandwiched between two semi-infinite, non-interacting leads without magnetic field. The leads mimic higher dimensional Fermi liquids.

tinger liquid parameter in the charge (spin) sector, u_c (u_s) is the associated velocity, while $\phi_{c,s}$ and $\theta_{c,s}$ are the bosonic charge and spin fields with $[\phi_i(x), \theta_j(x')] = \delta_{ij}\,(i\pi/2)\,\mathrm{sgn}(x'-x)$ $(i,j \in \{c,s\})$, and where $\phi_{c,s}$ ($\theta_{c,s}$) are proportional to the integrated charge and spin densities (currents). The applied magnetic field is denoted by B, and α is a short distance cutoff. Following Refs. [7–9], the coupling to higher-dimensional Fermi liquid leads is simulated by connecting the interacting Luttinger liquid to semi-infinite, non-interacting Luttinger liquids described by

$$H_L^< = \sum_{i=c,s} \frac{1}{2\pi} \int_{-\infty}^{-L/2} dx \left(v_F (\partial_x \phi_i)^2 + v_F (\partial_x \theta_i)^2 \right) \quad (2)$$

and an analogous expression $H_L^>$ for $x > L/2$. Being non-interacting, the leads have Luttinger liquid parameters $K_c = K_s = 1$, and all velocities equal the Fermi velocity v_F. This setup is sketched in Fig. 2.

Following Refs. [16], we first argue that the magnetic field B is relevant in the renormalization group (RG) sense, which leads to a gap for the combination of fields $\phi_c + \theta_s$ (corresponding to right-moving electrons with spin down and left-moving electrons with spin up). In a Rashba wire, this gap is precisely the one around zero momentum, see Fig. 1(*b*). [12] To see how interactions renormalize this gap, we derive the RG equation for the magnetic field in a real space RG analysis with running short distance cutoff $\alpha(b) = \alpha\, b$. The flow stops when $\alpha(b)$ equals the length scale related to the running gap. At a given RG stage, this gap can be defined by the expansion of the cosine term to second order (which is strictly speaking only justified at the end of the flow). [21] For frequencies much larger than both the temperature T and the finite size energy v_F/L, the RG equation of the magnetic field is given by the one of an infinite wire at zero temperature,

$$\frac{\partial B}{\partial \ln(b)} = \left(1 - \frac{K_c}{2} - \frac{1}{2K_s} \right) B \; . \quad (3)$$

In order to draw on this RG analysis, we are now looking for the strong coupling theory describing the low energy physics of the system, and thus search for a transformation that makes the gapped field $\phi_+ \sim \phi_c + \theta_s$ explicit. The second field ϕ_- deriving from this transformation, to which the magnetic field does not couple, will then describe the low energy physics in the strong coupling regime. This canonical transformation takes the general form

$$\phi_c = C_+\, A\, \phi_+ + C_-\, (1-A)\, \phi_- \; , \quad (4a)$$

$$\theta_c = \frac{1}{C_+}\, \theta_+ + \frac{1}{C_-}\, \theta_- \; , \quad (4b)$$

$$\phi_s = \frac{1}{C_+}\, \theta_+ - \frac{1}{C_-}\, \frac{A}{1-A}\, \theta_- \; , \quad (4c)$$

$$\theta_s = C_+\, (1-A)\, \phi_+ - C_-\, (1-A)\, \phi_- \; , \quad (4d)$$

where A, C_+ and C_- are arbitrary real numbers. While $C_\pm$ parametrizes the trivial rescaling $\phi_\pm \to C_\pm\, \phi_\pm$ and $\theta_\pm \to (1/C_\pm)\, \theta_\pm$, the parameter A allows for a non-trivial modification of the canonical transformation. It seems at first sight natural to use $A = 1/2$, which results in orthogonal fields, [21]

$$\phi_+ \sim \phi_c + \theta_s \quad , \qquad \phi_- \sim \phi_c - \theta_s \; . \quad (5)$$

Writing the electric current j as the imaginary time τ derivative of the spatially integrated charge density, [21]

$$j \sim \partial_\tau \phi_c \sim \partial_\tau \frac{\phi_+}{2} + \partial_\tau \frac{\phi_-}{2} \; , \quad (6)$$

we could now argue that the conductance equals $G = e^2/h$. Provided that $\phi_\pm$ as chosen in Eq. (5) describe the strong coupling regime, the first term of Eq. (6) vanishes, while the second one remains unaffected. Because the two terms are symmetric in the charge field ϕ_c, the conductance is reduced from $2\,e^2/h$ to $1\,e^2/h$. The use of chiral fields of given spin, $\phi_{r\sigma}$, defined by $\phi_{r\sigma} = r\phi_\sigma + \theta_\sigma$ with $\sigma = \uparrow, \downarrow$ and $r = R, L = \pm$, while $\phi_\sigma = (\phi_c \pm \phi_s)/\sqrt{2}$ and $\theta_\sigma = (\theta_c \pm \theta_s)/\sqrt{2}$, also corresponds to such an orthogonal choice. However, the fields ϕ_+ and ϕ_- only have to be linearly independent, not orthogonal. This is the reason why there is a non-trivial parameter A in the general transformation given in Eq. (4). Even more so, the very notion of orthogonality of $\phi_\pm$ is not particularly well defined. To see this, let us perform the trivial rescaling (with $a, b \in \mathbb{R}$)

$$\phi_c = \frac{1}{a}\, \phi'_c \; , \quad \theta_c = a\, \theta'_c \; , \quad \phi_s = \frac{1}{b}\, \phi'_s \; , \quad \theta_s = b\, \theta'_s \; . \quad (7)$$

This transformation does not change the form of the Hamiltonian, but simply modifies the Luttinger liquid parameters and velocities, and furthermore brings the sine-Gordon term to the form $\sim \cos\left(\sqrt{2}(\frac{1}{a}\, \phi'_c + b\, \theta'_s)\right)$. Using orthogonal fields would now lead to

$$\phi_- = b\,\phi_c' - \frac{1}{a}\,\theta_s' = ab\,\phi_c - \frac{1}{ab}\,\theta_s \ , \tag{8}$$

which manifestly differs from Eq. (5). The use of ϕ_- as given in Eq. (8) for the calculation of the current j along the lines of Eq. (6) would furthermore yield a current different from above. These inconsistencies underline that orthogonality is not a good criterion for the definition of $\phi_\pm$.

We will now show how renormalization group (RG) arguments can help to find the strong coupling theory. Going back to the general transformation given in Eq. (4), we note that different choices of A have a two-fold effect. First, the transformation introduces interactions (off-diagonal elements) of the form $(\partial_x\phi_+)\,(\partial_x\phi_-)$ and $(\partial_x\theta_+)\,(\partial_x\theta_-)$ with A-dependent interaction strengths. Second, the parameter A also determines the scaling dimension of the magnetic field with respect to the *diagonal* part of the transformed Hamiltonian. Of course, the full scaling dimension as defined in Eq. (3) follows from the full Hamiltonian, and is *not* affected by the canonical transformation given in Eq. (4). As a key result, we find that one can always choose A such that the full scaling dimension of the magnetic field is already obtained from the *diagonal* part of the Hamiltonian. [22] The off-diagonal terms do then not affect the strong coupling scaling of the magnetic field B and can be interpreted as only renormalizing the velocities and Luttinger liquid parameters. We therefore argue that this special choice of

$$A = \frac{K_c}{K_c + 1/K_s} \ . \tag{9}$$

yields the correct strong coupling degrees of freedom. For any other choice of A, in contrast, the RG marginal off-diagonal terms are essential for the scaling of the magnetic field B. Being off-diagonal, they imply that the chosen basis is not the appropriate one.

To now obtain the fully correct strong coupling theory, one would have to perform a coupled RG analysis of all velocities, Luttinger liquid parameters, off-diagonal terms and the magnetic field. This would be implemented by diagonalizing the quadratic part of the Hamiltonian at each RG step, deriving the corrections due to high-energy fluctuations, and iterating. [23] At a given step, the degrees of freedom appropriate for the description of the strong coupling physics at this step are the ones defined by A as given in Eq. (9), evaluated with scale dependent Luttinger liquid parameters (or, more generally, with whatever A chosen such that already the diagonal part of the transformed Hamiltonian yields the full scaling dimension of the magnetic field). This calculation is at least numerically possible. In our case, however, the RG analysis of the magnetic field alone, together with the choice of A given in Eq. (9) is a sufficiently good approximation to the strong coupling theory. As has already been discussed in Refs. [16], the RG flow of the magnetic field is rather short and the Luttinger liquid parameters and velocities are basically unmodified during this flow.

On a more fundamental level, we note that there also exists a special case in which our approximate treatment becomes exact. If the charge and spin velocities are equal, $u_c = u_s$, the choice of A given in Eq. (9), and only this choice, yields a diagonal Hamiltonian (see Eq. (12b) below). While experiments typically have $u_c \neq u_s$, this exactly solvable case underlines the existence of a non-trivial strong coupling theory. In addition, it has been shown in Refs. [23] that the velocities u_c and u_s approach each other during the RG flow (although, as mentioned above, the flow is cut off at a very early stage in the system considered here).

We thus proceed with the choice of Eq. (9). Using a convenient choice of $C_\pm$, the canonical transformation reads

$$\phi_c = \frac{K_c}{\sqrt{K}}\,\phi_+ + \sqrt{\frac{K_c}{K_s\,K}}\,\phi_- \ , \tag{10a}$$

$$\theta_c = \frac{1}{\sqrt{K}}\,\theta_+ + \frac{1}{\sqrt{K_c\,K_s\,K}}\,\theta_- \ , \tag{10b}$$

$$\phi_s = \frac{1}{\sqrt{K}}\,\theta_+ - \sqrt{\frac{K_s\,K_c}{K}}\,\theta_- \ , \tag{10c}$$

$$\theta_s = \frac{1}{K_s\sqrt{K}}\,\phi_+ - \sqrt{\frac{K_c}{K_s\,K}}\,\phi_- \ , \tag{10d}$$

where $K = K_c + 1/K_s$. Inside the wire, this transformation yields

$$\begin{aligned} H_W = \int_{-L/2}^{L/2} \frac{dx}{2\pi} \Big[& u_+(\partial_x\phi_+)^2 + u_+(\partial_x\theta_+)^2 \\ & + u_-(\partial_x\phi_-)^2 + u_-(\partial_x\theta_-)^2 \\ & + 2U\,\left((\partial_x\phi_+)(\partial_x\phi_-) + (\partial_x\theta_+)(\partial_x\theta_-)\right)\Big] \\ & + \frac{B}{2\pi\alpha}\int_{-L/2}^{L/2} dx\,\cos\left(\sqrt{2K}\phi_+\right) \ . \end{aligned} \tag{11}$$

with

$$u_+ = \frac{u_c K_c + u_s/K_s}{K} \ , \quad u_- = \frac{u_c/K_s + u_s\,K_c}{K} \ , \tag{12a}$$

$$U = \frac{u_c - u_s}{K}\sqrt{\frac{K_c}{K_s}} \ . \tag{12b}$$

We would like to note that this transformation has already been used in Refs. [16, 18, 19], however without pointing out its importance and implications. The partially gapped Hamiltonian describing the low energy physics in the wire can now be derived as outlined above

by first analyzing the effect of the magnetic field using RG, and then expanding the cosine term to second order. This effective strong coupling theory in turn allows us to calculate the conductance through the wire. In linear response, it is given by the Kubo formula

$$G = \frac{2e^2}{\pi^2}\,\omega_n \left.\langle \phi_c(x,\omega_n)\phi_c(x',-\omega_n)\rangle\right|_{i\omega_n\to\omega+i0^+,\ \omega\to 0}, \tag{13}$$

where ω_n are Matsubara frequencies. We are thus essentially left with the calculation of the propagator of the charge field ϕ_c. Using Eq. (10), the conductance can be decomposed into the new degrees of freedom $\phi_\pm$,

$$G = G_{++} + G_{+-} + G_{-+} + G_{--}\ , \tag{14}$$

where the contributions $G_{ij} = \lim_{\omega\to 0} G_{ij}^{i\omega_n\to\omega+i0^+}$ follow from

$$\begin{aligned}
G_{++}^{i\omega_n} &= \frac{K_c^2}{K}\,\frac{2e^2}{\pi^2}\,\omega_n\,\langle \phi_+(x,\omega_n)\phi_+(x',-\omega_n)\rangle\ , \qquad (15)\\
G_{+-}^{i\omega_n} &= \frac{K_c}{K}\sqrt{\frac{K_c}{K_s}}\,\frac{2e^2}{\pi^2}\,\omega_n\,\langle \phi_+(x,\omega_n)\phi_-(x',-\omega_n)\rangle\ ,\\
G_{-+}^{i\omega_n} &= \frac{K_c}{K}\sqrt{\frac{K_c}{K_s}}\,\frac{2e^2}{\pi^2}\,\omega_n\,\langle \phi_-(x,\omega_n)\phi_+(x',-\omega_n)\rangle\ ,\\
G_{--}^{i\omega_n} &= \frac{K_c}{K_s K}\,\frac{2e^2}{\pi^2}\,\omega_n\,\langle \phi_-(x,\omega_n)\phi_-(x',-\omega_n)\rangle\ .
\end{aligned}$$

Because the system is inhomogeneous, the propagators on the right-hand side of Eq. (15) are defined as the Green's functions of the real space action. In the leads, the differential equations associated with these Green's functions can conveniently be inferred from the ones defining the propagators of the initial spin and charge fields by virtue of Eq. (10). This procedure is detailed in the supplementary material. [24] Inside the wire, the Hamiltonian can be simplified in a mean-field approach, which allows to drop off-diagonal terms $\sim (\partial_x\phi_+)(\partial_x\phi_-)$ and $\sim (\partial_\tau\phi_+)(\partial_\tau\phi_-)$ involving derivatives of the gapped field ϕ_+, leading to particularly simple differential equations for the propagators in this region. The derivation of the matching conditions at the boundaries and the final integration of the differential equations involve some lengthy algebra. The latter is sketched in the supplementary material [24] and follows along the lines of Refs. [7–9, 25]. We find that only the conductance G_{--} associated with the gapless mode ϕ_- is non-zero, whereas the conductance G_{++} as well as the mixed conductances $G_{\pm\mp}$ vanish, in accordance with the gap of ϕ_+,

$$G_{++} = G_{+-} = G_{-+} = 0\ , \tag{16a}$$

$$G_{--} = \frac{e^2}{\pi}\,\frac{K_c^2 + 1/K_s^2}{K^2}\ . \tag{16b}$$

Restoring $\hbar = h/(2\pi) = 1$, we thus obtain the total conductance of a spiral Luttinger liquid as

$$G = \sum_{ij} G_{ij} = G_{--} = \frac{e^2}{h}\,\frac{2\,K_c^2\,K_s^2 + 2}{(K_c\,K_s + 1)^2}\ . \tag{17}$$

In the non-interacting case $K_c = K_s = 1$ as well as the helical limit $K_c = 1/K_s$, the conductance has the expected universal value $G = G_0 = e^2/h$. In the general case, however, the conductance takes a non-universal value depending on the interactions inside the wire, even in the presence of Fermi liquid leads. For gated semiconductor wires, one may use $K_c = 0.65$ and $K_s = 1$, [26–28] which yields a conductance of $G = 1.045\,e^2/h$. Wires with strong, long range (unscreened) Coulomb repulsion, on the other hand, allow for Luttinger parameters as low as $K_c = 0.2$. [29–33] In the quasi-helical regime, this would result in a conductance of $G = 1.4\,e^2/h$. Physically, the non-universal value of the conductance may be interpreted as follows. The conductance of a spinful quantum wire connected to Fermi liquid leads can in general be written as

$$G = K_L\,\frac{2e^2}{h}\,\mathcal{O}\ , \tag{18}$$

where $K_L = 1$ is the charge Luttinger liquid parameter in the leads, [7–9] and where $\mathcal{O}$ is the overlap of the charge mode with the gapless mode ϕ_-. In this language, a Mott insulator has for instance $\mathcal{O} = 0$. There is however no restriction of the overlap to $\mathcal{O} = 0$ (Mott insulator), $\mathcal{O} = 1/2$ (helical Luttinger liquid) or $\mathcal{O} = 1$ (spin density wave state). If the interactions of ϕ_c are different from the interactions of θ_s, i.e. for $K_c \neq 1/K_s$, it is natural that the charge and spin degrees of freedom are not equally favored to have gaps, and that the system analyzed in this work will contain more gapped charge or more gapped spin, depending on the ratio of K_c and $1/K_s$. We would also like to stress that the non-universal character of the conductance is here obtained by a different mechanism than in an infinite helical Luttinger liquid, where $G = K_c\,e^2/h$. [5, 6] In the latter case, an entire helical (charge) mode is gapless, but interactions hinder the electrical current and therefore render the conductance non-universal. Here, the charge moves as if is was non-interacting (because $K_L = 1$), but it is not half of the total charge that moves.

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Supplementary material for "Non-universal conductance in quasi-helical quantum wires"

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In this supplementary material, we give some details on the derivation of the propagators of the degrees of freedom $\phi_\pm$ describing the partially gapped regime.

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GENERAL FORM OF THE ACTION AND THE PROPAGATORS

We analyze an interacting quantum wire in a partially gapped, quasi-helical regime connected to two Fermi liquid leads, see Fig. 3. This system is modeled by a Hamiltonian $H = H_L^< + H_W + H_L^>$. The wire region is described by

$$\begin{aligned} H_W = &\frac{1}{2\pi}\int_{-L/2}^{L/2} dx \left(\frac{u_c}{K_c}(\partial_x\phi_c)^2 + u_c K_c(\partial_x\theta_c)^2\right) \\ &+ \frac{1}{2\pi}\int_{-L/2}^{L/2} dx \left(\frac{u_s}{K_s}(\partial_x\phi_s)^2 + u_s K_s(\partial_x\theta_s)^2\right) \\ &+ \frac{B}{2\pi\alpha}\int_{-L/2}^{L/2} dx\, \cos\left(\sqrt{2}(\phi_c+\theta_s)\right) \ , \end{aligned} \tag{19}$$

while the leads are modeled as

$$H_L^< = \sum_{i=c,s}\frac{1}{2\pi}\int_{-\infty}^{-L/2} dx \left(v_F(\partial_x\phi_i)^2 + v_F(\partial_x\theta_i)^2\right) \ , \tag{20}$$

and an analogous expression for $x > L/2$. A motivation for these expressions, along with a definition of the various symbols, is given in the main text. [1] Next, we apply the canonical transformation given in Eq. (10) of the main text. [1] This brings the Hamiltonian to the form

$$\begin{aligned} H = \int_{-\infty}^{\infty}\frac{dx}{2\pi}\Big[&\frac{u_+(x)}{K_+(x)}(\partial_x\phi_+)^2 + u_+(x)K_+(x)\,(\partial_x\theta_+)^2 \\ &+ \frac{u_-(x)}{K_-(x)}(\partial_x\phi_-)^2 + u_-(x)K_-(x)\,(\partial_x\theta_-)^2 \\ &+ 2U_\phi(x)\,(\partial_x\phi_+)(\partial_x\phi_-) + 2U_\theta(x)\,(\partial_x\theta_+)(\partial_x\theta_-) \\ &+ \frac{B(x)}{2\pi\alpha}\cos\left(\sqrt{2}\,\phi_+\right)\Big] \ . \end{aligned} \tag{21}$$

As explained in the main text, [1] we can now treat the sine-Gordon term by a renormalization group (RG) analysis. Inside the wire, the sine-Gordon term flows to strong coupling. This in turn gaps out the field ϕ_+. In the leads, which are non-interacting and have $B(x) = 0$, the RG analysis reduces the cutoff of the theory, but does not open up any gaps. At the end of the flow, the sine-Gordon term can be expanded to second order, which defines the gap of ϕ_+ in the wire.

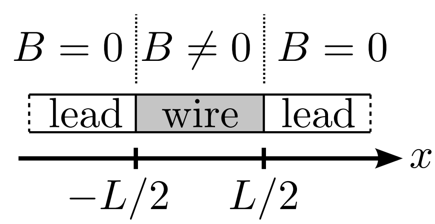

FIG. 3: Sketch of the system considered: an interacting wire subject to a field B is sandwiched between two semi-infinite, non-interacting leads without magnetic field. The leads mimic higher dimensional Fermi liquids.

Next, we switch from the Hamiltonian to the associated imaginary time action and integrate out the fields $\theta_\pm$. After Fourier transformation from imaginary time to bosonic Matsubara frequencies ω_n and a partial integration in real space, we obtain the action

$$\mathcal{S} = \frac{1}{2}\sum_{\omega_n}\int_{-\infty}^{\infty} dx\, \mathbf{\Psi}(x,\omega_n)^T\, \hat{D}_{\omega_n}(x)\, \mathbf{\Psi}(x,-\omega_n) \tag{22}$$

where $\mathbf{\Psi}(x,\omega_n)^T = (\phi_+(x,\omega_n), \phi_-(x,\omega_n))$, and where the matrix differential operator $\hat{D}_{\omega_n}(x)$ has the form

$$\begin{aligned} \hat{D}_{\omega_n}(x) = &\begin{pmatrix} A(x,\omega_n) & B(x,\omega_n) \\ C(x,\omega_n) & D(x,\omega_n)\end{pmatrix} \\ &- \partial_x \begin{pmatrix}\tilde{A}(x) & \tilde{B}(x) \\ \tilde{C}(x) & \tilde{D}(x)\end{pmatrix}\partial_x \ . \end{aligned} \tag{23}$$

The matrix elements of $\hat{D}_{\omega_n}(x)$ (to be detailed below) depend on Matsubara frequency and position. Within either the leads or the wire, they are constant in space.

At the interfaces, however, they jump. The propagators of the fields $\phi_\pm$ can now be defined as the Green's function of the matrix operator $\hat{D}_{\omega_n}(x)$. To this end, we rewrite the action as

$$\mathcal{S} = \frac{1}{2}\sum_{\omega_n} \int dx \int dx' \, \mathbf{\Psi}(x,\omega_n)^T \, \hat{D}_{\omega_n}(x,x') \, \mathbf{\Psi}(x',-\omega_n) \ , \tag{24}$$

where

$$\hat{D}_{\omega_n}(x,x') = \delta(x-x') \, \hat{D}_{\omega_n}(x) \ . \tag{25}$$

The matrix Green's function associated with the differential operator $\hat{D}_{\omega_n}(x,x')$ is then given by

$$\begin{aligned} \mathcal{G}_{\omega_n}(x',x) &= \langle \mathbf{\Psi}(x',-\omega_n) \mathbf{\Psi}(x,\omega_n)^T \rangle \qquad (26) \\ &= \begin{pmatrix} \mathcal{G}^{++}_{\omega_n}(x',x) & \mathcal{G}^{-+}_{\omega_n}(x',x) \\ \mathcal{G}^{+-}_{\omega_n}(x',x) & \mathcal{G}^{--}_{\omega_n}(x',x) \end{pmatrix} \ , \end{aligned}$$

with

$$\mathcal{G}^{ij}_{\omega_n}(x',x) = \langle \phi_i(x',-\omega_n) \phi_j(x,\omega_n) \rangle \ . \tag{27}$$

By definition, it satisfies the differential equation [2]

$$\int_{-\infty}^{\infty} dx'' \, \hat{D}_{\omega_n}(x,x'') \, \mathcal{G}_{\omega_n}(x'',x') = \delta(x-x') \, \mathbb{1} \ , \tag{28}$$

and thus

$$\hat{D}_{\omega_n}(x) \, \mathcal{G}_{\omega_n}(x,x') = \delta(x-x') \, \mathbb{1} \ . \tag{29}$$

DEFINING THE PROPAGATORS IN THE WIRE

Inside the wire, the field ϕ_+ is gapped and pinned to one of the minima of the cosine in Eq. (19). Fluctuations around its mean field value can to a good approximation be neglected if one is interested in the behavior of system at lowest energies (and thus especially for the calculation of the conductance in linear response). We hence drop terms $\sim (\partial_x\phi_+)(\partial_x\phi_-)$ and $\sim (\partial_\tau\phi_+)(\partial_\tau\phi_-)$ inside the wire, which yields

$$\mathcal{S}_W = \frac{1}{2}\sum_{\omega_n} \int_{-L/2}^{L/2} dx \, \mathbf{\Psi}(x,\omega_n)^T \, \hat{D}^W_{\omega_n}(x) \, \mathbf{\Psi}(x,\omega_n) \ , \tag{30}$$

where $\hat{D}^W_{\omega_n}(x)$ has the diagonal form

$$\hat{D}^W_{\omega_n}(x) = \begin{pmatrix} \hat{\mathcal{D}}^+_{\omega_n}(x) & 0 \\ 0 & \hat{\mathcal{D}}^-_{\omega_n}(x) \end{pmatrix} \tag{31}$$

with

$$\hat{\mathcal{D}}^+_{\omega_n}(x) = \frac{\omega_n^2 + \Delta^2}{\pi u_+^* K_+^*} - \partial_x\left(\frac{u_+^*}{\pi K_+^*}\partial_x\right) \ , \tag{32}$$

$$\hat{\mathcal{D}}^-_{\omega_n}(x) = \frac{\omega_n^2}{\pi u_-^* K_-^*} - \partial_x\left(\frac{u_-^*}{\pi K_-^*}\partial_x\right) \ . \tag{33}$$

Here, $u_\pm^*$ and $K_\pm^*$ are the effective velocities and Luttinger liquid parameters obtained after the RG flow and the integration over the fields $\theta_\pm$, while Δ denotes the gap of ϕ_+. The precise values of these quantities are however not important for the conductance once the Fermi liquid leads are taken into account (see below). [3]

We now proceed to the integration of the equations for the off-diagonal Green's functions. For x inside the wire, these equations read

$$\hat{\mathcal{D}}^-_{\omega_n}(x) \, \mathcal{G}^{+-}_{\omega_n}(x,x') = 0 \ , \quad \hat{\mathcal{D}}^+_{\omega_n}(x) \, \mathcal{G}^{-+}_{\omega_n}(x,x') = 0 \ . \tag{34}$$

Because the off-diagonal Green's functions furthermore satisfy

$$\mathcal{G}^{+-}_{\omega_n}(x,x') = \mathcal{G}^{-+}_{-\omega_n}(x',x) \ , \tag{35}$$

we find that for x and x' inside the wire, the only possible solutions are

$$\mathcal{G}^{+-}_{\omega_n}(x,x') = \mathcal{G}^{-+}_{\omega_n}(x,x') = 0 \ . \tag{36}$$

This result expresses that a ϕ_+ excitation can not be transformed into a ϕ_- excitation inside the wire (at least in the mean field approximation leading to the diagonal action in Eq. (31)).

The diagonal propagators, again for x in the wire, are defined by

$$\hat{\mathcal{D}}^+_{\omega_n}(x) \, \mathcal{G}^{++}_{\omega_n}(x,x') = \delta(x-x') \ , \tag{37}$$

$$\hat{\mathcal{D}}^-_{\omega_n}(x) \, \mathcal{G}^{--}_{\omega_n}(x,x') = \delta(x-x') \ . \tag{38}$$

Integrating these equations (see below) leaves us with two constants, which have to be determined by boundary conditions at $\pm L/2$. These conditions encode the matching of the propagators and their derivatives at the interfaces.

DEFINING THE PROPAGATORS IN THE LEADS

In order to determine the boundary conditions, we first derive the differential equation $\hat{D}_{\omega_n}(x) \, \mathcal{G}_{\omega_n}(x,x')$ defining the propagators for x in the leads. There, the natural degrees of freedom are not the transformed fields, $\phi_\pm$, but rather the initial ones, ϕ_c and θ_s. In terms of these fields, the action in the leads is diagonal and reads

$$\mathcal{S}_L^< = \frac{1}{2}\sum_{\omega_n}\int_{-\infty}^{-L/2} dx\, \mathbf{\Phi}(x,\omega_n)^T\, \hat{D}^L_{\omega_n}(x)\, \mathbf{\Phi}(x,\omega_n) \ , \tag{39}$$

where $\mathbf{\Phi}(x,\omega_n)^T = (\phi_c(x,\omega_n), \theta_s(x,\omega_n))$, and with

$$\hat{D}^L_{\omega_n}(x) = \begin{pmatrix} \hat{\mathcal{D}}_{\omega_n}(x) & 0 \\ 0 & \hat{\mathcal{D}}_{\omega_n}(x) \end{pmatrix} \ , \tag{40}$$

$$\hat{\mathcal{D}}_{\omega_n}(x) = \frac{\omega_n^2}{\pi v_F} - \partial_x(\frac{v_F}{\pi}\partial_x) \ . \tag{41}$$

This defines the spin and charge propagators by the differential equations

$$\hat{\mathcal{D}}_{\omega_n}(x)\langle\phi_c(x,-\omega_n)\phi_c(x',\omega_n)\rangle = \delta(x-x') \ , \tag{42}$$

$$\hat{\mathcal{D}}_{\omega_n}(x)\langle\theta_s(x,-\omega_n)\theta_s(x',\omega_n)\rangle = \delta(x-x') \ , \tag{43}$$

while the mixed propagators obey

$$\hat{\mathcal{D}}_{\omega_n}(x)\langle\phi_c(x,-\omega_n)\theta_s(x',\omega_n)\rangle = 0 \ , \tag{44}$$

$$\hat{\mathcal{D}}_{\omega_n}(x)\langle\theta_s(x,-\omega_n)\phi_c(x',\omega_n)\rangle = 0 \ . \tag{45}$$

We can now use the definition of the canonical transformation from ϕ_c and θ_s to $\phi_\pm$,

$$\phi_c = \frac{K_c}{\sqrt{K}}\,\phi_+ + \sqrt{\frac{K_c}{K_s\,K}}\,\phi_- \ , \tag{46a}$$

$$\theta_s = \frac{1}{K_s\sqrt{K}}\,\phi_+ - \sqrt{\frac{K_c}{K_s\,K}}\,\phi_- \ , \tag{46b}$$

with $K = K_c + 1/K_s$, to obtain coupled differential equations for the propagators of $\phi_\pm$. These may be recast into the matrix form

$$\hat{D}_{\omega_n}(x)\,\mathcal{G}_{\omega_n}(x,x') = \delta(x-x')\,\mathbb{1} \tag{47}$$

with

$$\hat{D}_{\omega_n}(x) = \begin{pmatrix} \frac{K_c^2+1/K_s^2}{K} & \frac{K_c-1/K_s}{K}\sqrt{\frac{K_c}{K_s}} \\ \frac{K_c-1/K_s}{K}\sqrt{\frac{K_c}{K_s}} & \frac{2K_c}{K_sK} \end{pmatrix}\,\hat{\mathcal{D}}_{\omega_n}(x) \ . \tag{48}$$

The exact same result can of course be obtained by first transforming the lead degrees of freedom from $\phi_{c,s}$ and $\theta_{c,s}$ to $\phi_\pm$ and $\theta_\pm$, and then integrating out the fields $\theta_\pm$. Equation (48) furthermore implies the differential equations

$$\hat{\mathcal{D}}^+_{\omega_n}(x)\,\mathcal{G}^{++}_{\omega_n}(x,x') = \delta(x-x') \ , \tag{49}$$

$$\hat{\mathcal{D}}^-_{\omega_n}(x)\,\mathcal{G}^{--}_{\omega_n}(x,x') = \delta(x-x') \ , \tag{50}$$

where the differential operators $\hat{\mathcal{D}}^\pm_{\omega_n}(x)$ read (for x in the leads)

$$\hat{\mathcal{D}}^+_{\omega_n}(x) = \frac{\omega_n^2}{\pi v_F K_{+,L}} - \partial_x(\frac{v_F}{\pi K_{+,L}}\partial_x) \ , \tag{51}$$

$$\hat{\mathcal{D}}^-_{\omega_n}(x) = \frac{\omega_n^2}{\pi v_F K_{-,L}} - \partial_x(\frac{v_F}{\pi K_{-,L}}\partial_x) \ , \tag{52}$$

with

$$K_{+,L} = \frac{2}{K} \ , \tag{53}$$

$$K_{-,L} = \frac{K_s}{K_cK}\left(K_c^2 + \frac{1}{K_s^2}\right) \ . \tag{54}$$

BOUNDARY CONDITIONS AND SOLUTION

The comparison of Eqs. (31) and (48) with the Eq. (23) allows to determine the matrix elements of the general differential equation (23) away from the interfaces at $x = \pm L/2$. At these latter points, the boundary conditions for the propagators follow from integrating Eq. (23) over the interface. Since we will ultimately be interested in the propagators $\mathcal{G}^{ij}_{\omega_n}(x,x')$ with both x and x' inside the wire, we fix x' to some position inside the wire from now on. To derive the boundary conditions with respect to x at $\pm L/2$, we first note that the general equation (23) implies

$$\hat{D}^{11}_{\omega_n}(x)\mathcal{G}^{++}_{\omega_n}(x,x') = \delta(x-x') - \hat{D}^{12}_{\omega_n}(x)\mathcal{G}^{+-}_{\omega_n}(x,x') \ , \tag{55}$$

$$\hat{D}^{21}_{\omega_n}(x)\mathcal{G}^{++}_{\omega_n}(x,x') = -\hat{D}^{22}_{\omega_n}(x)\mathcal{G}^{+-}_{\omega_n}(x,x') \ , \tag{56}$$

where $\hat{D}^{ij}_{\omega_n}(x)$ are the matrix elements of $\hat{D}_{\omega_n}(x)$ as given in Eq. (23). The boundary conditions for $\mathcal{G}^{++}_{\omega_n}(x,x')$ at $x_0 = -L/2$ can now be found by integrating the expression

$$\left(\hat{D}^{11}_{\omega_n}(x) - \frac{\tilde{B}(x_0^-)}{\tilde{D}(x_0^-)}\hat{D}^{21}_{\omega_n}(x)\right)\mathcal{G}^{++}_{\omega_n}(x,x') \tag{57}$$

from $x_0^- = -L/2 - \epsilon$ to $x_0^+ = -L/2 + \epsilon$ with a small $\epsilon > 0$ and taking the limit $\epsilon \to 0$. Using Eqs. (55) and (56), and given that the propagators are continuous, the terms not involving derivatives drop out, and we obtain the boundary condition

$$\begin{aligned} &\tilde{A}(x_0^+)\,\partial_x\mathcal{G}^{++}_{\omega_n}(x_0^+,x') \\ &= \left(\tilde{A}(x_0^-) - \frac{\tilde{B}(x_0^-)\tilde{C}(x_0^-)}{\tilde{D}(x_0^-)}\right)\,\partial_x\mathcal{G}^{++}_{\omega_n}(x_0^-,x') \ . \end{aligned} \tag{58}$$

In the derivation of this equation, a term involving $\mathcal{G}^{+-}_{\omega_n}(x_0^-,x')$ has canceled trivially, while another one involving $\mathcal{G}^{+-}_{\omega_n}(x_0^+,x')$ is zero because the off-diagonal propagators vanish when both $x=x_0^+$ and x' are inside the wire, see Eq. (36). In the first line of Eq. (58), we furthermore used $\tilde{C}(x_0^+,\omega_n)=0$ according to Eq. (31). With the definitions of $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ and $\tilde{D}$ from Eqs. (31) and (48), we can finally bring the boundary condition for the propagator $\mathcal{G}^{++}_{\omega_n}$ at $x_0=-L/2$ to the form

$$\frac{u_+^*}{K_+^*}\,\partial_x\mathcal{G}^{++}_{\omega_n}(x_0^+,x')=\frac{v_F}{K_{+,L}}\,\partial_x\mathcal{G}^{++}_{\omega_n}(x_0^-,x')\ . \tag{59}$$

In the same way, we can derive the boundary condition at $x_1=L/2$. With $x_1^\pm=x_1\pm\epsilon$, we obtain

$$\frac{u_+^*}{K_+^*}\,\partial_x\mathcal{G}^{++}_{\omega_n}(x_1^-,x')=\frac{v_F}{K_{+,L}}\,\partial_x\mathcal{G}^{++}_{\omega_n}(x_1^+,x')\ . \tag{60}$$

These boundary conditions, together with Eqs. (37) and (51), allow to understand the propagator $\mathcal{G}^{++}_{\omega_n}(x,x')$ as the solution of the differential equation (valid for x' inside the wire and any x)

$$\hat{\mathcal{D}}^+_{\omega_n}(x)\,\mathcal{G}^{++}_{\omega_n}(x,x')=\delta(x-x') \tag{61}$$

where

$$\hat{\mathcal{D}}^+_{\omega_n}(x)=\left[\frac{\omega_n^2}{\pi u_+(x)K_+(x)}-\partial_x\left(\frac{u_+(x)}{\pi K_+(x)}\right)\partial_x\right]\ , \tag{62}$$

and where

$$u_+(x)=\begin{cases}v_F & x\in\ \text{leads}\\ u_+^*\frac{\omega_n}{\sqrt{\omega_n^2+\Delta^2}} & x\in\ \text{wire}\end{cases}\ , \tag{63}$$

$$K_+(x)=\begin{cases}K_{+,L}=2/K & x\in\ \text{leads}\\ K_+^*\frac{\omega_n}{\sqrt{\omega_n^2+\Delta^2}} & x\in\ \text{wire}\end{cases}\ . \tag{64}$$

An analogous calculation can be performed for $\mathcal{G}^{--}_{\omega_n}$. We find that this propagator obeys the equation (also defined for x' inside the wire and any x)

$$\hat{\mathcal{D}}^-_{\omega_n}(x)\,\mathcal{G}^{--}_{\omega_n}(x,x')=\delta(x-x') \tag{65}$$

where

$$\hat{\mathcal{D}}^-_{\omega_n}(x)=\left[\frac{\omega_n^2}{\pi u_-(x)K_-(x)}-\partial_x\left(\frac{u_-(x)}{\pi K_-(x)}\right)\partial_x\right]\ , \tag{66}$$

and where

$$u_-(x)=\begin{cases}v_F & x\in\ \text{leads}\\ u_-^* & x\in\ \text{wire}\end{cases}\ , \tag{67}$$

$$K_-(x)=\begin{cases}K_{-,L}=\frac{K_s}{K_cK}\left(K_c^2+1/K_s^2\right) & x\in\ \text{leads}\\ K_-^* & x\in\ \text{wire}\end{cases}\ . \tag{68}$$

The boundary conditions at $x_0=-L/2$ read

$$\frac{u_-^*}{K_-^*}\,\partial_x\mathcal{G}^{--}_{\omega_n}(x_0^+,x')=\frac{v_F}{K_{-,L}}\,\partial_x\mathcal{G}^{--}_{\omega_n}(x_0^-,x')\ , \tag{69}$$

while they are given by

$$\frac{u_-^*}{K_-^*}\,\partial_x\mathcal{G}^{--}_{\omega_n}(x_1^-,x')=\frac{v_F}{K_{-,L}}\,\partial_x\mathcal{G}^{--}_{\omega_n}(x_1^+,x') \tag{70}$$

at $x_1=L/2$. We are now finally in the position to actually calculate the propagators $\mathcal{G}^{++}_{\omega_n}(x,x')$ and $\mathcal{G}^{--}_{\omega_n}(x,x')$, which has, for instance, already been done in Refs. [3, 4]. After some lengthy but straightforward algebra, we find

$$\mathcal{G}^{ii}_{\omega_n}(x,x')=\frac{\pi K_{i,W}}{2|\omega_n|}\,e^{-|\omega_n|\frac{|x-x'|}{u_{i,W}}}+\frac{\pi K_{i,W}}{|\omega_n|}\,\frac{(\kappa_i^-)^2e^{-|\omega_n|\frac{L}{u_{i,W}}}\,\cosh\left(|\omega_n|\frac{x-x'}{u_{i,W}}\right)+\kappa_i^-\kappa_i^+\,\cosh\left(|\omega_n|\frac{x+x'}{u_{i,W}}\right)}{(\kappa_i^+)^2e^{|\omega_n|\frac{L}{u_{i,W}}}-(\kappa_i^-)^2e^{-|\omega_n|\frac{L}{u_{i,W}}}}\ , \tag{71}$$

where $\kappa_i^\pm=1/K_{i,W}\pm1/K_{i,L}$ with $K_{i,W}$ being the value of $K_i(x)$ inside the wire and $K_{i,L}$ being the value of $K_i(x)$ inside the leads, and where $u_{i,W}$ is the value of $u_i(x)$ inside the wire (note that $K_{+,W}\neq K_+^*$ and $u_{+,W}\neq u_+^*$). The conductance, being proportional the zero frequency limits of $\omega_n\,\mathcal{G}^{ij}_{\omega_n}(x,x')$ for x and x' inside the wire, can now be checked to be independent of the effective velocities u_i^* and Luttinger liquid parameters K_i^* in the wire. Also, we note that $\mathcal{G}^{--}_{\omega_n}=\mathcal{O}(\omega_n^{-1})$, while $\mathcal{G}^{++}_{\omega_n}=\mathcal{O}(\omega_n^0)$ and $\mathcal{G}^{-+}_{\omega_n}=\mathcal{G}^{+-}_{\omega_n}=0$ (for x and x' inside the wire). Therefore, only the term $\sim\omega_n\,\mathcal{G}^{--}_{\omega_n}$ contributes to the conductance.

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